



Descripción del modelo

Tema-1b



Remembering



An atom is classified according to its number of protons and neutrons:

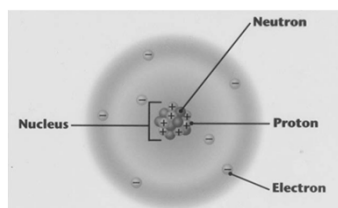
The number of protons in the nucleus of an atom determines an element's atomic number "chemical element"

All carbon atoms, and only carbon atoms, contain six protons and have an atomic number of 6

The number of neutrons determines the "isotope" of that element. All atoms have a mass number, which is the sum of protons and neutrons.

The carbon atom has several isotopes. The most abundant with six neutrons and one with seven neutrons (^{12}C and ^{13}C)

The nuclei of all atoms may be characterized by:
a nuclear spin quantum number (I)



Only nuclei with **spin number (I) $\neq 0$** can absorb/emit electromagnetic radiation.



NMR Active nucleus



The nuclear spin quantum number I can either be equal to zero, or to multiples of $1/2$

For atoms with there is no nuclear spin and therefore, they cannot have a nuclear magnetic resonance. These atoms are called NMR silent. All other values of I yield nuclear spin.



Even mass # and Even atomic #
 $I=0$ (^{12}C , ^{16}O , etc)

No Nuclear spin
NMR Inactive

Odd mass# and Even atomic #
 $I=1/2$ (^1H , ^{13}C , ^{15}N)

Nuclear Spin
 Spherical charge distribution

Odd mass# and Odd atomic#
 $I= n/2$ ($n \neq 1$) integer $I=3/2$ (^{11}B , ^{23}Na); ^{53}Ca $I=7/2$

Ellipsoidal charge distribution

Even mass # and Odd atomic #
 $I=\text{whole integer}$ $I=1$ (^2H , ^{14}N) ; $I=3$ ^{10}B



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Tabla periódica



Tabla periódica de isótopos de RMN.

1																	18																		
$1/2$ H																	3 He																		
6/7 Li	9 Be															13 B	14 C	15 N	16 O	17 F	18 Ne														
23 Na	24 Mg	3 Al	4 Si	5 P	6 S	7 Cl	8 Ar											19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
39 K	40 Ca	41 Sc	42 Ti	43 V	44 Cr	45 Mn	46 Fe	47 Co	48 Ni	49 Cu	50 Zn	51 Ga	52 Ge	53 As	54 Se	55 Br	56 Kr	87 Rb	88 Sr	89 Y	90 Zr	91 Nb	92 Mo	93 Tc	94 Ru	95 Rh	96 Pd	97 Ag	98 Cd	99 In	100 Sn	101 Sb	102 Te	103 I	104 Xe
55 Cs	56 Ba	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	85 At	86 Rn	119 Fr	120 Ra	121 Ac	122 Th	123 Pa	124 U	125 Np	126 Pu	127 Am	128 Cm	129 Bk	130 Cf	131 Es	132 Fm	133 Md	134 No	135 Lr
111 Ds	112 Nh	113 Nh	114 Nh	115 Nh	116 Nh	117 Nh	118 Nh	119 Nh	120 Nh	121 Nh	122 Nh	123 Nh	124 Nh	125 Nh	126 Nh	127 Nh	128 Nh	129 Nh	130 Nh	131 Nh	132 Nh	133 Nh	134 Nh	135 Nh	136 Nh	137 Nh	138 Nh	139 Nh	140 Nh	141 Nh	142 Nh	143 Nh	144 Nh	145 Nh	146 Nh

Espin nuclear = I										
$1/2$	1	$3/2$	$5/2$	3	$7/2$	4	$9/2$	5	6	7



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Nuclear properties related to NMR



Nuclear properties

Nuclide	Spin I	Electric quadrupole moment ^{a)} [eQ] [10 ⁻²⁸ m ²]	Natural abundance [%]	Relative sensitivity ^{b)}	Gyromagnetic ratio $\gamma^{a)}$ [10 ⁷ rad T ⁻¹ s ⁻¹]	NMR frequency [MHz] ^{b)} ($B_0 = 2.3488$ T)
¹ H	1/2	-	99.985	1.00	26.7519	100.0
² H	1	2.87 x 10 ⁻³	0.015	9.65 x 10 ³	4.1066	15.351
³ H ^{c)}	1/2	-	-	1.21	28.5350	106.664
⁶ Li	1	-6.4 x 10 ⁻⁴	7.42	8.5 x 10 ⁻³	3.9371	14.716
¹⁰ B	3	8.5 x 10 ⁻²	19.58	1.99 x 10 ⁻²	2.8747	10.746
¹¹ B	3/2	4.1 x 10 ⁻²	80.42	0.17	8.5847	32.084
¹² C	0	-	98.9	-	-	-
¹³ C	1/2	-	1.108	1.59 x 10 ⁻²	6.7283	25.144
¹⁴ N	1	1.67 x 10 ⁻²	99.63	1.01 x 10 ⁻³	1.9338	7.224
¹⁵ N	1/2	-	0.37	1.04 x 10 ⁻³	-2.7126	10.133
¹⁶ O	0	-	99.96	-	-	-
¹⁷ O	5/2	-2.6 x 10 ⁻²	0.037	2.91 x 10 ⁻²	-3.6280	13.557
¹⁹ F	1/2	-	100	0.83	25.1815	94.077
²³ Na	3/2	0.1	100	9.25 x 10 ⁻²	7.0704	26.451
²⁵ Mg	5/2	0.22	10.13	2.67 x 10 ⁻³	-1.6389	6.1195
²⁸ Si	1/2	-	4.70	7.84 x 10 ⁻³	-5.3190	19.865
³¹ P	1/2	-	100	6.63 x 10 ⁻²	10.8394	40.481
³⁹ K	3/2	5.5 x 10 ⁻²	93.1	5.08 x 10 ⁻⁴	1.2499	4.667
⁴³ Ca	7/2	-5.0 x 10 ⁻²	0.145	6.40 x 10 ⁻³	-1.8028	6.728
⁵⁷ Fe	1/2	-	2.19	3.37 x 10 ⁻⁵	0.8687	3.231
⁵⁹ Co	7/2	0.42	100	0.28	6.3015	23.614
¹¹⁹ Sn	1/2	-	8.58	5.18 x 10 ⁻²	-10.0318	37.272
¹³³ Cs	7/2	-3.0 x 10 ⁻³	100	4.74 x 10 ⁻²	3.5339	13.117
¹⁹⁵ Pt	1/2	-	33.8	9.94 x 10 ⁻³	5.8383	21.499

NMR Periodic Table

http://www-usr.rider.edu/~grushow/nmr/NMR_tutor/periodic_table/nmr_pt_frameset.html

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Nuclear spin and Magnetic Moment



Nucleus rotates about its axis (spin)

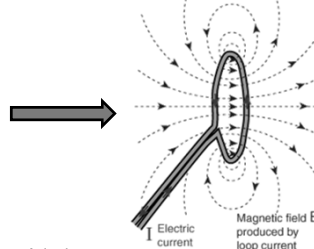
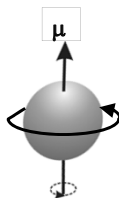
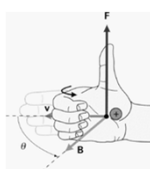
Nuclear spin results in angular momentum (\mathbf{P}). $\mathbf{P} = \hbar \times \sqrt{I(I+1)}$

Quantized spin quantum number I
 $2I+1$ States $I, I-1, I-2, \dots, -I$

Since the nucleus is charged, spin will produce a magnetic momentum (\mathbf{u})

$$\boldsymbol{\mu} = \gamma \mathbf{P}$$

Where γ is the proportionality constant called the gyromagnetic ratio



Similar to magnetic field created by electric current flowing in a coil

The magnetic moment is quantized (m)

for proton $m = +1/2$ & $-1/2$

$$m = I, I-1, I-2, \dots, -I$$

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Quantization of magnetic moment



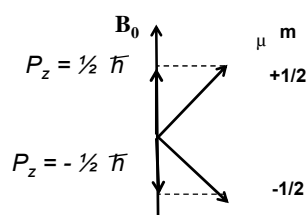
The angular momentum for a nucleus, in a static magnetic field, will be oriented directionally

$$P_z = m\hbar$$

m is the magnetic quantum number $m = I, I-1, I-2, \dots, -I$

therefore, there are $(2I+1)$ values for m , and $(2I+1)$ possible orientations for the angular momentum

For a nucleus with $I = 1/2$ there are two possible orientations of the magnetic moment



This is the simplest case only two orientations and only one transition between them

the components of the magnetic moment along z

$$\mu_z = \gamma P_z = m\gamma\hbar$$



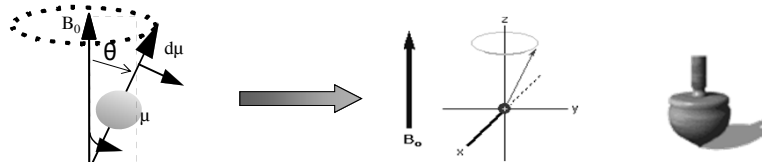
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Precession Classical description



In a external magnetic field the nuclear spin precesses at Larmor frequency

$$\omega_0 = -\gamma B_0 \text{ (rad s}^{-1}\text{)} \xrightarrow{\nu = \omega_0 / 2\pi} \nu_0 = \gamma B_0 / 2\pi \text{ (Hz)}$$



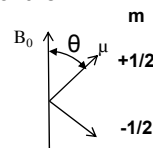
Larmor frequency related with γ and B_0

The relative orientation of the magnetic moment (θ) is depend of value I


For a nucleus with $I = 1/2$ there are two possible orientations of the magnetic moment

(General rule: # orientations = $2I+1$):


This is the simplest case, only two orientations and only one transition between them



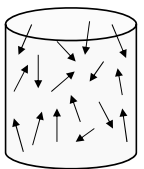
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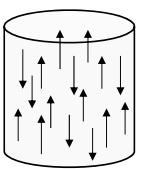
Magnetic alignment



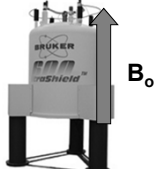
$\uparrow = \gamma \hbar / 4\pi$



In the absence of external field, each nuclei is energetically degenerate



Add a strong external field (B_0) and the nuclear magnetic moment: aligns with (low energy) against (high-energy)




B_0

At equilibrium (in the magnetic field), there is excess of nuclei in a state of low energy

The energy of a spin in a magnetic field (E) will depend on a static magnetic field called B_0 , and μ .

$$E = -\mu_z B_0 = -m\gamma\hbar B_0$$




For $l=1/2$ then m can be $+1/2$ and $-1/2$


$m = -1/2$ (β) ——— $E_\beta = +1/2\gamma\hbar B_0$

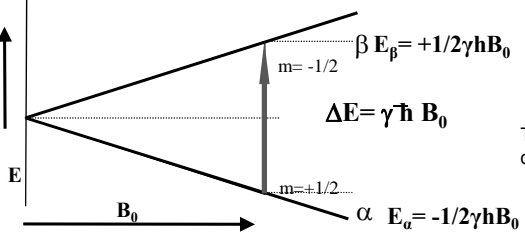
$m = +1/2$ (α) ——— $E_\alpha = -1/2\gamma\hbar B_0$

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Nuclear Energy levels in a Static Magnetic Field





$E = -\mu_z B_0 = -m\gamma\hbar B_0$

The energy difference is linearly dependent on γ and on B_0

- At thermal equilibrium the energy difference between α and β states prevents these states from being equally populated
- The relative population of a particular state is given by the Boltzman distribution:

$$\frac{N_\beta}{N_\alpha} \approx 1 - \frac{\gamma \hbar B_0}{K_B T}$$

$B_0 = 1.41\text{T (60 MHz)} \quad \frac{N_\beta}{N_\alpha} = 0.9999904$

$B_0 = 1.41\text{T (800 MHz)} \quad \frac{N_\beta}{N_\alpha} = 0.999987$

N_α = number of nuclei in the α state

N_β = number of nuclei in the β state

K_B = Boltzman constant $K_B = 1.3085 \times 10^{-23} \text{ JK}^{-1}$

T = temperature (Kelvin)

The Difference is very small (ppm)

NMR IS INSENSITIVE

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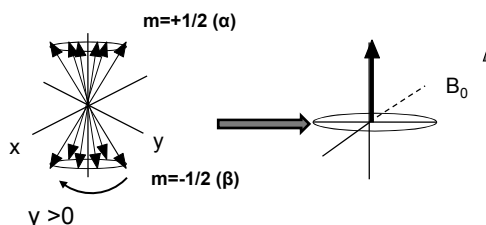
Net Magnetization



Classic vectorial View:

- Nuclei either align with or against external magnetic field along the z-axis.

- Since more nuclei align with field, net magnetization (M_0) exists parallel to external magnetic field

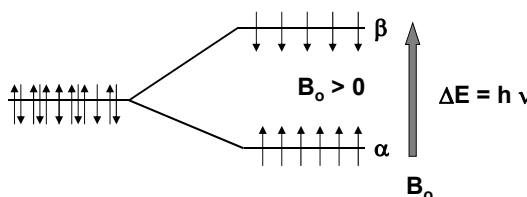


Quantum Description:

- Nuclei either populate low energy (α , aligned with field) or high energy (β , aligned against field)

- Net population in a lower energy level.

- Absorption of radio-frequency promotes nuclear spins from $\alpha \rightarrow \beta$.



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Resonance



The relationship between the Larmor frequency and ΔE is as follows:

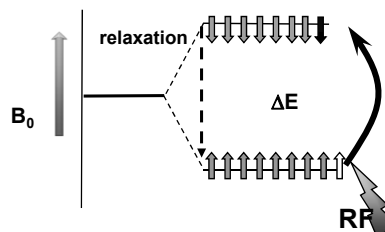
$$\Delta E = \hbar \nu_i \quad \Delta E = \gamma \hbar B_0 \quad \text{and} \quad \nu_L = (\gamma/2\pi) B_0, \quad \Delta E = h \nu_L$$

Transitions between energy levels (between α and β spin states for spin 1/2 nuclei) are quantized, and can only be promoted by an energy ΔE

In NMR the transitions are promoted by an applied electromagnetic field, B_1 with a frequency, ν_i (radiofrequency) matching the Larmor frequency of the nucleus

$$\nu_L = \nu_i = (\gamma/2\pi) B_0 \quad \Delta E = h \nu_i$$

This is known as **resonance** ($\nu_i = \nu_L$), when the frequency of our externally applied electromagnetic field (B_1) are coincident with the Larmor frequency of the nucleus of interest



$$N_\alpha > N_\beta \quad \xrightarrow{\nu_i = \nu_L} \quad \text{signal}$$

$$N_\alpha = N_\beta \quad \xrightarrow{\quad \quad \quad} \quad \text{Saturation}$$

<http://ochem.jsd.claremont.edu/movies.dir/nmr.htm>

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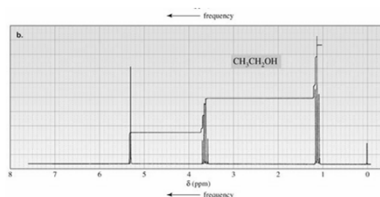
Continuous Wave (CW)



NMR can be performed like other spectroscopy's (UV/vis, IR) by simply slowly varying the frequency of monochromatic incident radiation and monitoring for absorption

The first spectrometers using a frequency sweep, a constant magnetic field, to obtain the spectra.

One alternative was to sweep the magnetic field while maintaining a constant frequency.



The scan rate determines the resolution and the possibility of saturation of the signals. The average duration of a sweep might be between 2 and 10 minutes.

The system is very inefficient because much of the time was recorded only noise in the spectrum.

Most spectrometers could not add different sweeps

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Fourier Transform



How to efficiently detect a range (spectrum) of NMR frequencies



4.3 SENSITIVITY OF FOURIER SPECTROSCOPY 157

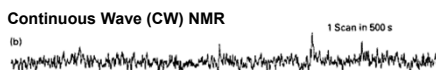
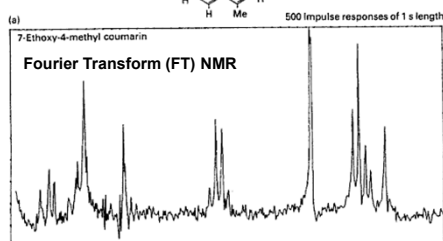
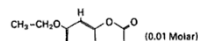


FIG. 4.3.4. 60-MHz proton magnetic resonance spectra of 7-ethoxy-4-methyl coumarin. (a) Fourier transform of 500 free induction signals recorded in 500 s. (b) Single scan recorded in 500 s by slow passage on the same instrument. (Reproduced from Ref. 4.130.)

R.R Ernst et al Principles NMR

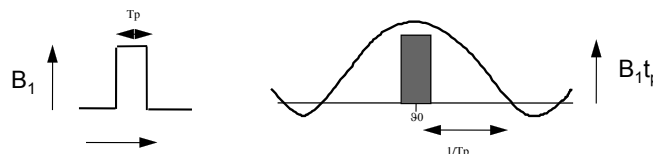
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Pulse NMR



In pulsed Fourier transform NMR, for a given nucleus (^1H for example), all frequencies are excited simultaneously by a short, high power radiofrequency pulse (B_1 field)



The pulse is applied at a particular frequency, ν_1 , but a short pulse excites a large continuous band of frequencies (the **bandwidth**) centered around ν_1 .

The useful or effective bandwidth is proportional to $1/\tau_p$ (τ_p is the **pulse length**, also called the **pulse width** or **pulse duration**).

The **pulse amplitude** is a measure of the power with which the pulse is applied, and determines the strength of the B_1 field



τ_p is usually very short, i.e. μs

For all frequencies can be excited in a homogeneous mode must be satisfied that $\gamma B_1 = 2\pi SW$, (SW equal to spectral window), $PW_{90} \ll 1/4sw$

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Rotating Frame



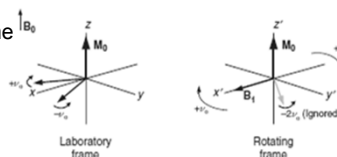
The RF pulse provides an oscillating magnetic field B_1 (at Larmor Frequency) in transverse plane (is equivalent to two counter-rotating vectors)



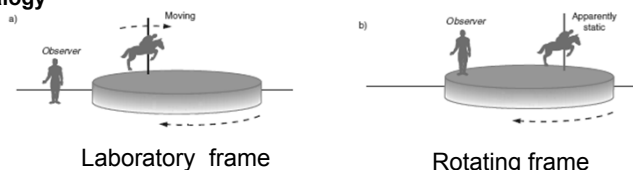
Simplification: Laboratory frame to Rotating Frame

The system rotate to the equal RF frequency ν_0

The $-2\nu_0$ is far to the resonance frequency and may be ignored




Analogy

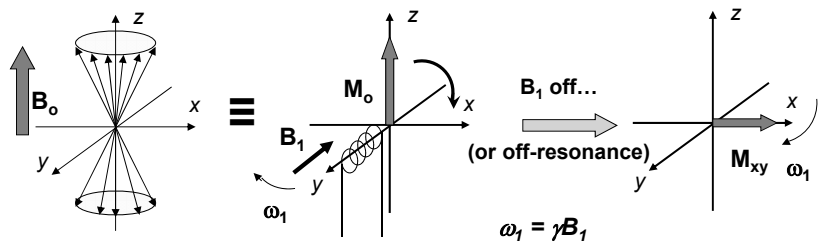


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RMN Experiment

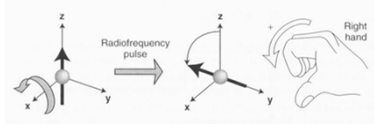


Resonant condition: frequency (ω_1) of B_1 matches Larmor frequency (ω_0) energy is absorbed and population of α and β states are perturbed.

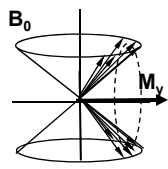


$\omega_1 = \gamma B_1$

M_0 now precesses about B_1 for as long as the B_1 field is applied.




Right-hand rule



Phase coherence

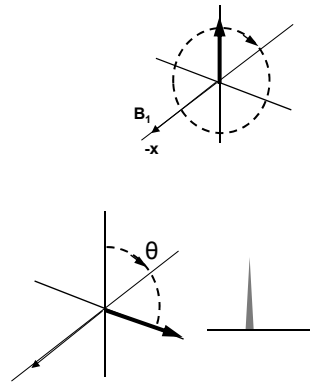
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Pulse effect

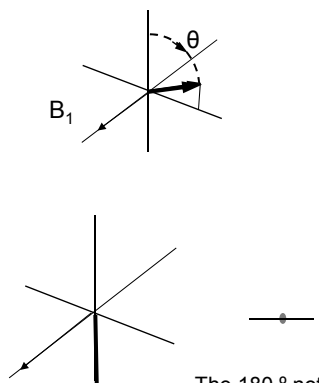


The pulse generates M_{xy} transverse magnetization

The flip angle is: $\alpha = 360 \gamma B_1 t_p$ degrees



The 90° pulse correspond to maximum signal



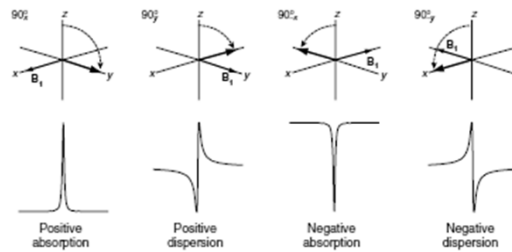
The 180° not produce signal (saturation)

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Pulse phase



Detection in -y



Some sequences use a different phase pulse: Phase Cycling
Selecting the some signals in NMR experiment and rejecting
the those that are not required



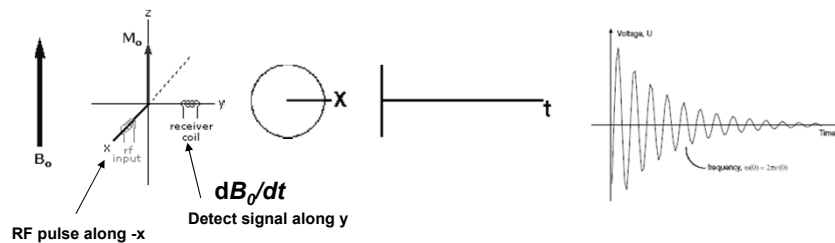
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NMR Signal Detection FID: Free Induction Decay



The pulse generates M_{xy} transverse magnetization that precesses around the z axis at precession frequency ω_0

The FID reflects the change in the magnitude of M_{xy} as the signal is changing relative to the receiver along the y-axis



Again, the signal is precessing about B_0 at its Larmor Frequency (ω_0).



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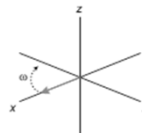
Events after the pulse: Evolution of the magnetization



The magnetic moment (M) precessing about a static magnetic field (B_0) results in a local magnetic field (B') varying in time ($dM/dt \propto dB'/dt$)



The X & Y components
(Free Induction Decay):

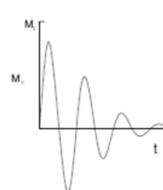
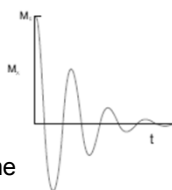


$$M_y(t) = -M_0 \cos(\omega_0 t) e^{-t/T_2}$$

$$M_x(t) = M_0 \sin(\omega_0 t) e^{-t/T_2}$$

$$M_x = M_0 \cos \omega t e^{-t/T_2}$$

$$M_y = M_0 \sin \omega t e^{-t/T_2}$$



T_2 transverse relaxation time



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NMR Sensitivity in NMR



The sensitivity of a nucleus depends of :

- Gyromagnetic constant
- External magnetic field
- Natural abundance isotope to observe

$$dM/dt \propto \gamma B_0 M \propto N \gamma^3 B_0^2 h^2 I(I+1) / (3k_B T)$$

Isotope	I	γ ($10^7 \text{ rad T}^{-1} \text{ s}^{-1}$)	Abundancia N(%)	Resonance Frec. B=2.3488T	relative* sensitivity
^1H	1/2	26.7519	99.98	100.0	1
^{19}F	1/2	25.1815	100	94.077	0.83
^{31}P	1/2	10.8394	100	40.481	6.63×10^{-2}
^{13}C	1/2	6.7283	1.10	25.144	1.56×10^{-2}
^2H	1	4.1066	0.015	15.351	9.65×10^{-3}
^{15}N	1/2	-2.7126	0.37	10.133	1.04×10^{-3}

$$\gamma \text{ } ^1\text{H} = 26,753 \text{ rad/G}$$

$$\gamma \text{ } ^{13}\text{C} = 6,728 \text{ rad/G}$$

$$\text{Ratio } (\gamma \text{ } ^1\text{H} / \gamma \text{ } ^{13}\text{C})^3 \approx 64$$

If we consider the term A (Natural abundance) $^1\text{H} \approx 100\%$; $^{13}\text{C} \approx 1\%$

^1H is 6400 times more sensible than ^{13}C

Nuclei with larger γ will absorb/emit more energy, and will therefore be more sensitive.



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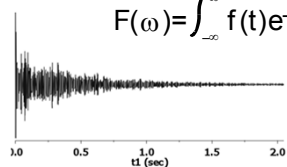
Time Domain to Frequency Domain Fourier Transform



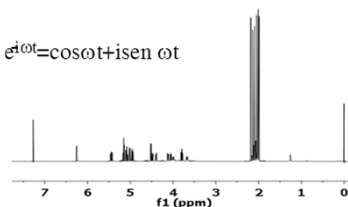
Time domain signals are converted into frequency domain signals using the **Fourier Transform**



$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$



$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

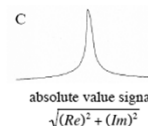
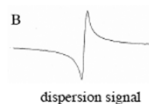
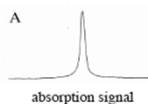


$f(t)$ corresponds to the time domain, and $F(\omega)$ corresponds to the frequency domain

$F(\omega)$ is a complex function that has a real (Re) and an imaginary part (Im)

Re
Im

Absorption
Dispersion

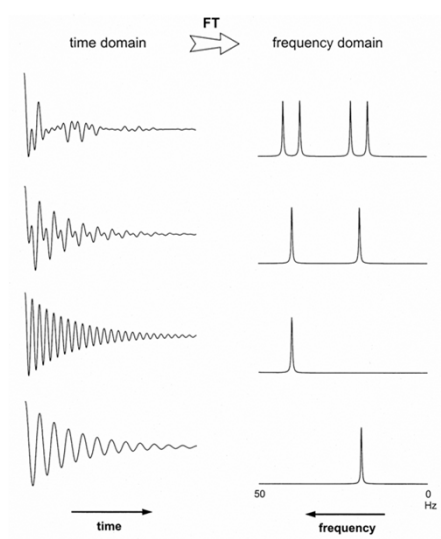


line shape is Lorentzian (Fourier transform of a decaying exponential function)



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Frequency Domain



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